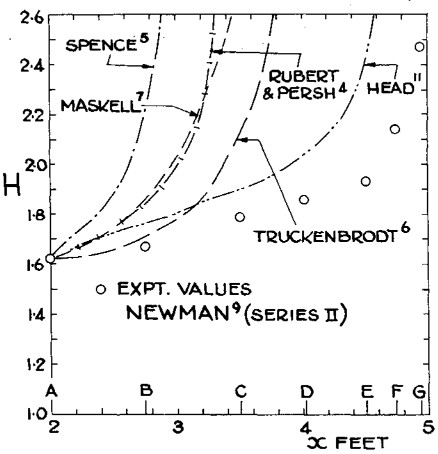

Fig. 4 Comparisons of calculated shape-factor developments.

For the calculation of shape-factor (H) development, different methods[4–7, 11] give widely differing results, as indicated in Figs. 3–5 using the measured momentum thickness development, in each case, as the basis for the calculations.

The generally rather poor predictions of some of the better known methods is clearly shown by these typical comparisons using data from Refs. 8–10. It will be seen, however, that the method of Head[11] gives the best over-all agreement with experiment and, consequently, it is recommended that this method should be used for the calculation of H, together with the momentum integral equation for the prediction of θ, the two equations being solved simultaneously step-by-step.

It is hoped that a more complete account of this work and a somewhat improved calculation method will be published at a later date.

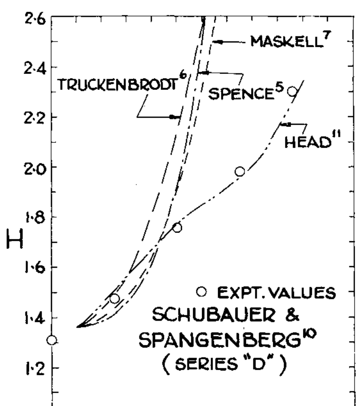

Fig. 5 Comparisons of calculated shape-factor developments.

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An Aerofoil Probe for Measuring the Transverse Component of Turbulence

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I. Introduction

At present, the accepted method of measuring instantaneously the transverse or v component of turbulent velocity employs the familiar crossed-wire probe in conjunction with dual channel hot-wire anemometer circuitry. Alternatively, for investigations where only root-mean-square values are required, a single slant wire can be used by rotating through 180° between measurements. Both of these methods involve expensive equipment and have several shortcomings. For two-point correlation work in particular, instantaneous values of v are required, which introduces the need for two crossed-wire probes and the accompanying four channels of electronics.

In an effort to circumvent this complexity a new and relatively simple probe has been developed at the Institute for Aerospace Studies. It is used with an ordinary inexpensive audio frequency amplifier. The probe consists basically of a small aerofoil and a force transducer that yields a voltage varying as the instantaneous value of v. More specifically, the aerofoil (of rectangular or circular planform) experiences a randomly varying lifting force, because of turbulent fluctuations in the flow. The aerofoil is attached to a tapered cantilever beam in which is imbedded a piezoelectric transducing element. For low values of turbulence intensity (i.e., less than 30%) the piezoelectric element produces an output voltage directly proportional to the v component of turbulent velocity.

II. Basic Theory

Figure 1 illustrates the basic principle of the aerofoil probe. We consider flow incident on the aerofoil with velocity V, at angle of attack α. In turbulent flow, V and α both vary in a random fashion. It is assumed that at any instant of time we can apply the approximation of quasi-steady linear aerofoil theory, provided that the frequency is not too high:

$$L = \tfrac{1}{2}\rho V^2 S[dC_L/d\alpha]\alpha \tag{1}$$

Received November 2, 1964.

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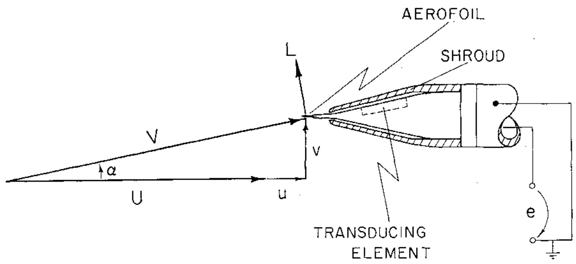

Fig. 1 Mechanism of response of aerofoil probe to v component of turbulence.

To a consistent approximation, α may be replaced by (v/U) and V by U, to give

$$L = \tfrac{1}{2}\rho US[dC_L/d\alpha]v = Ke \qquad (2)$$

Thus, in the range that the over-all probe response is linear in the lift L, the result is a voltage e proportional to v. (The proportionality constant includes and thus varies with the local stream velocity U.)

III. Probe Design

In the development of a satisfactory probe design several problems were encountered, the most significant being that of obtaining an over-all structural response free from resonant peaks over the frequency range of interest.

The probe to be described was developed for the study of turbulence in a 4-in. low-speed free airjet facility. With this facility frequencies from zero to approximately 10 kc/sec contribute significantly to the frequency spectrum. Consequently, it was desired that the aerofoil-cantilever portion of the beam have a first critical natural frequency somewhat higher than 10 kc. After considerable analysis and experiment it was found that a conical aluminum beam with a short outer section of uniform diameter (supporting the aerofoil sensor) would afford a first critical frequency of approximately 12.5 kc. This sufficiently satisfied the particular requirements.

In order to avoid sensitivity loss at the higher frequencies, it is desirable to use a lift-sensor of the smallest possible physical dimensions. This avoids spatial averaging of the short wave length turbulence components. On the other hand, it is desirable to have the maximum possible lifting area in order to obtain sufficient magnitudes of lift force and consequent output voltage. As a compromise, it was found that disks of 0.070 in. diam were small enough to give satisfactory high-frequency response, and large enough for good signal-to-noise ratio in the output amplifier. Rectangular planforms of comparable area and $AR = 2$ have been used with equal success.

Of necessity, a shroud was employed to insure that only the outer portion of the cantilever beam (primarily the sensing aerofoil) responds to the turbulent impulses. Figure 2 is a photograph of the complete probe assembly, both with the shroud in place and removed to expose the transducing element.

Another significant problem concerns accelerometric effects on the probe response. Vibrations of the probe support give rise to unwanted inertia loading on the cantilever beam, which seriously distorts the frequency spectrum of the output signal. By proper support design and the use of aluminum throughout the probe head, this problem was eliminated.

A ceramic PZT-4‡ piezoelectric element was used for the particular probe discussed. It deformed in the length expander mode. In a stream velocity U of 90 fps, with 10% turbulence, output voltages of the order of 5 mv were realized.

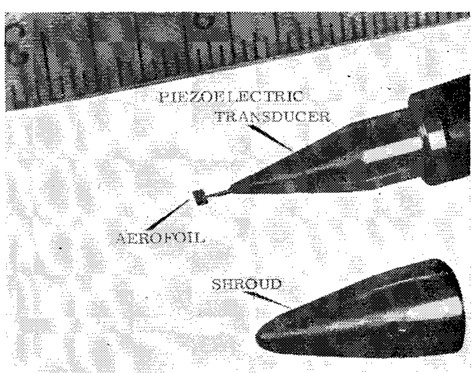

Fig. 2a Aerofoil probe for measuring the v component of turbulence (with shroud removed).

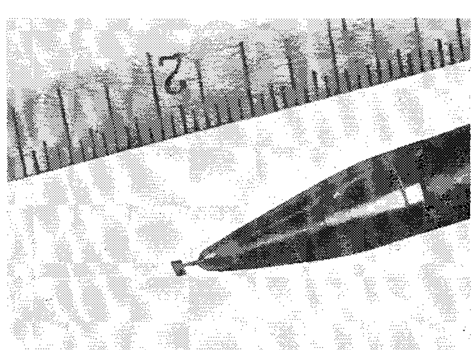

Fig. 2b Aerofoil probe with shroud in place.

IV. Experimental Results

Figure 3 compares frequency spectra obtained with the aerofoil probe and with a crossed-wire probe, for a common point in the jet flow. The curves are plotted in decibels relative to the 1000-cps point. Agreement between the two sets of data is excellent for frequencies between 40 and 4000 cps. Above 4000 cps the response begins to fall off as a result of the finite aerofoil size. A slight resonant peak is noted at 12.5 kc/sec.

The behavior at frequencies below 100 cps was found to be quite sensitive to the input impedance of the probe output amplifier. This was traced to the fact that the piezoelectric element behaves primarily like a voltage source in series with its own characteristic capacitance[1, 2] (i.e., about 130 mmf for the present probe). Serious response fall-off below 100 cps was obtained with an amplifier having a resistive input of 10 megohms. The insertion of a parallel capacitor-resistor pair (C = 25 mmf, R = 66 megohms) in series with the input resistor resulted in the much improved low-frequency response of Fig. 3.

Over the major frequency range then, the aerofoil probe appears to reproduce the v component frequency spectrum quite accurately. In addition, comparisons of root mean square velocity measurements obtained with the aerofoil probe and hot wires indicate that the probe output varies directly with turbulence velocity for intensities (i.e., v_{rms}/U) up to approximately 30%, as had been anticipated.

V. Concluding Remarks

Although the aerofoil probe is as yet only in the very early stages of exploitation, it would appear to have advantages for turbulence measurements in a number of circumstances. It provides a single channel, direct measurement of v, in contradistinction to the crossed-wire technique. A 90° axial rotation will yield a quick comparison of v and w as a check on turbulence axisymmetry. The obvious simplicity of the device points to ease of manufacture at an economical cost.

‡ Trade name of Clevite Corporation, Bedford, Ohio.

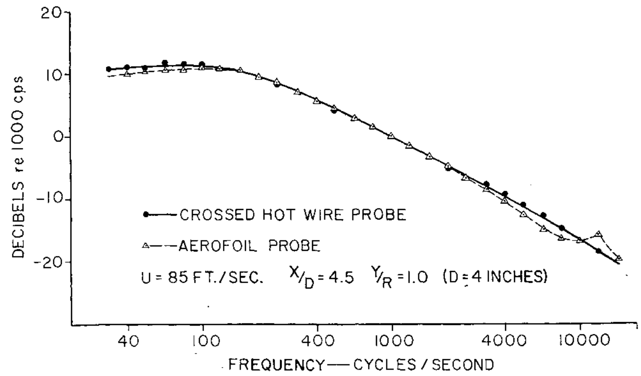

Fig. 3 Comparison of v component spectra in a round jet.

Superior durability and temporal stability should make possible measurements which were hitherto difficult with conventional crossed-wires (i.e., high velocities, dirty flows, hydrodynamic turbulence). Work is presently underway in the development of precise calibration techniques for the probe.

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Jet Damping of Symmetric Rockets

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Nomenclature

$\mathbf{H}$	= moment of momentum about center of mass
I_λ	= moment of inertia about the longitudinal axis
I_τ	= moment of inertia about any transverse axis
k	= radius of gyration
l	= distance from center of mass to jet along λ axis
m	= mass of rockets and contents at time t
$\mathbf{M}$, M	= moment, magnitude of moment
t	= time
XYZ	= inertial coordinates
α	= angle of attack
ρ	= radical distance to jet from λ axis
τ, ν, λ	= cross-spin coordinates
θ_p	= half angle of precession
θ, ϕ, ψ	= Euler angles
ω, Ω	= magnitude of angular velocity
$\mathbf{\Omega}$	= angular velocity of cross-spin coordinates

In recent years, the effect of jet damping on a spinning rocket has received considerable attention. Most studies have used the modified Eulerian dynamic equations[1] as the governing equations of motion. To find dynamic variables, such as transverse angular velocity and angle of attack, it is conventional to create complex expressions. There are exceptions,[2] but these have normally introduced complicated mathematical manipulations. Alternate equations of motion, derived herein, allow certain problems to be solved without the conventional complex variable substitution.

Received October 20, 1964.

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Assumptions

The equations of motion for a symmetric spinning body used by Nidey and Seames[3] are extended to include jet damping terms. In addition to symmetry, the conventional restrictions are assumed[4]: 1) no ejected particle is given an angular velocity relative to the rocket, 2) the angular momentum imparted to the particles is symmetric relative to the longitudinal axis, and 3) the center of mass always lies on the longitudinal axis. These assumptions give the general equation for rate of change of angular momentum:

$$\mathbf{M} = \dot{\mathbf{H}} + \text{rate of angular momentum transfer from the system} \tag{1}$$

Equations of Motion

Consider the cross-spin coordinate system τ, ν, λ shown in Fig. 1, where $\boldsymbol{\lambda}$ and $\boldsymbol{\tau}$ are unit vectors parallel and normal to the axis of symmetry, respectively. Let $\boldsymbol{\tau}$ be oriented so that the angular velocity of the rocket has only two components ω_λ and ω_τ. The transverse component of angular velocity is sometimes called cross-spin. Because the rocket and the coordinate system share the transverse angular velocity, the angular velocity of the coordinate system $\mathbf{\Omega}$ is written as

$$\mathbf{\Omega} = \Omega_\lambda \boldsymbol{\lambda} + \omega_\tau \boldsymbol{\tau} \tag{2}$$

For a symmetric rigid body, ignoring jet damping, the moment of momentum and its time derivative are

$$\mathbf{H} = I_\lambda \omega_\lambda \boldsymbol{\lambda} + I_\tau \omega_\tau \boldsymbol{\tau} \tag{3}$$

$$\dot{\mathbf{H}} = I_\lambda \dot{\omega}_\lambda \boldsymbol{\lambda} + I_\tau \dot{\omega}_\tau \boldsymbol{\tau} + I_\lambda \omega_\lambda (\mathbf{\Omega} \times \boldsymbol{\lambda}) + I_\tau \omega_\tau (\mathbf{\Omega} \times \boldsymbol{\tau}) \tag{4}$$

thus

$$M_\lambda = I_\lambda \dot{\omega}_\lambda \tag{5}$$

$$M_\tau = I_\tau \dot{\omega}_\tau \tag{6}$$

and

$$M_\nu = I_\tau \omega_\tau \Omega_\lambda - I_\lambda \omega_\lambda \omega_\tau \tag{7}$$

where $\boldsymbol{\nu} \equiv \boldsymbol{\lambda} \times \boldsymbol{\tau}$. The rate of rotation of the cross-spin (transverse angular velocity) about the spin axis obtained by rearranging Eq. (7) is

$$\Omega_\lambda = \Omega_0 + M_\nu/(I_\tau \omega_\tau) \tag{8}$$

where

$$\Omega_0 \equiv (I_\lambda/I_\tau)\omega_\lambda$$

To include the effects of jet damping it is only necessary to add additional terms to the right side of Eqs. (5) and (6).

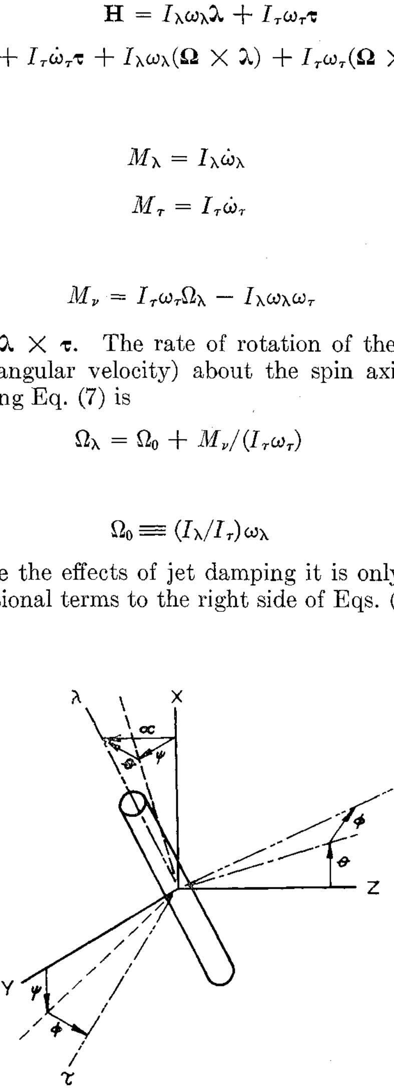

Fig. 1 Coordinates and Euler angles.